IEGR 350: Engineering Economy
Spring 2016
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Assignment 4

80 Points (Due: 5:00PM Wednesday Feb. 24)

Assessment Goals: (Interpolation, Geometric gradient series; Rate of return; Calculation accuracy)

Show your work. No round down or up, use 2 decimals for dollar values and 4 decimals for factors. Use tables from your textbook only, no formula.

PROBLEM:

For a project with annual compounding interest rate of 11.82%, the following information is provided:

$4,800 initial investment
$3,980 investment in year 1, decreasing by $550 each year through year 5
$3,509 annual withdrawal between years 4 through 9
$15,085 withdrawal in year 11, decreasing by 16% annually through year 18
$1,848 investment in year 14
$19,870 withdrawal in year 20

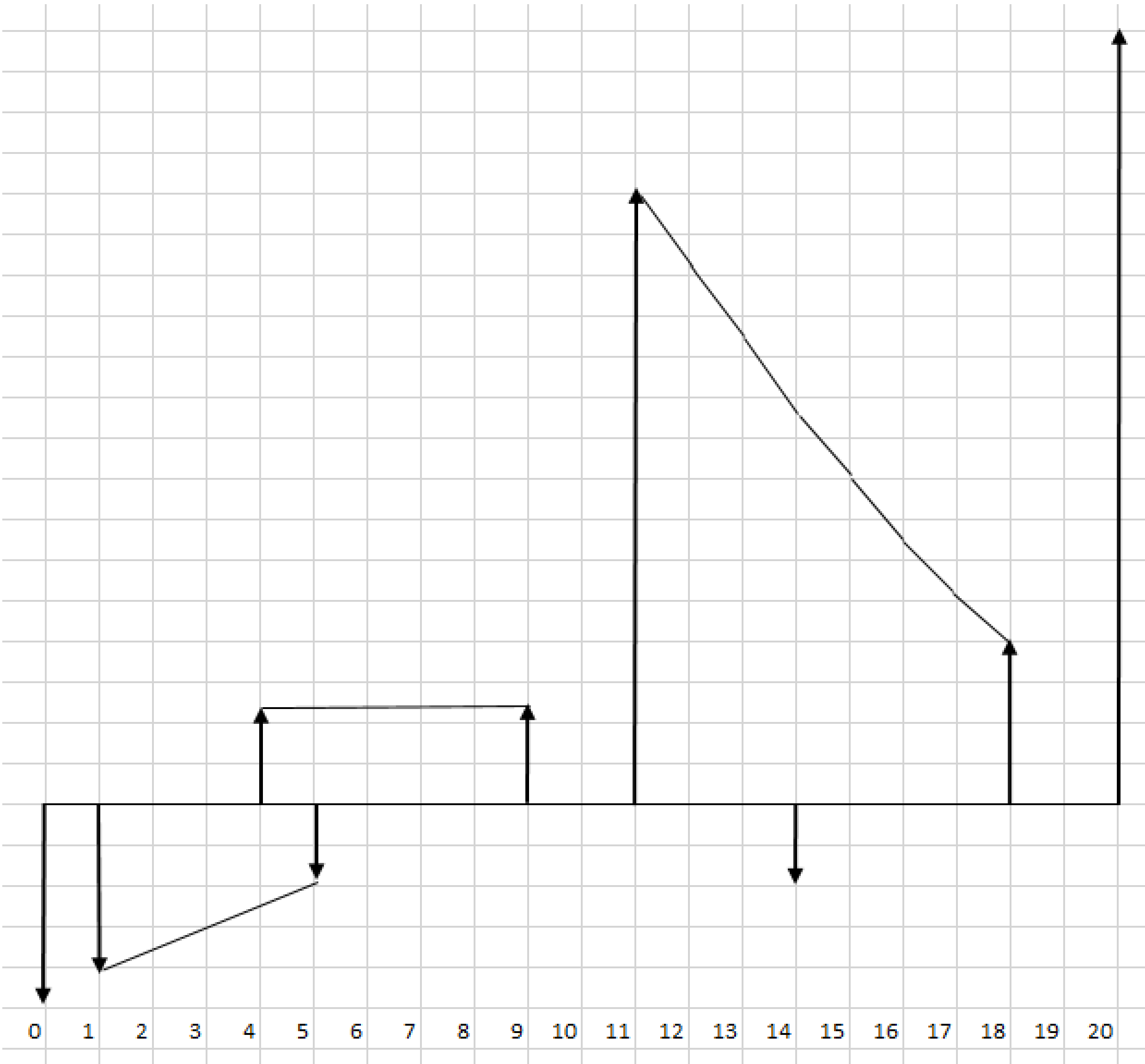
(1) Find out the future worth of the cash flow at the end of the project (30 pts)

(2) What is the geometric gradient series between years 4 to 12 with 10% annual increase that is equivalent to the project? (20 pts)

(3) What is the real rate of return of the project? (30 pts)

SOLUTION KEY:

First we draw the cash flow based on the given information, then make a decision of what approach to take to solve the problem. Part one requires a future worth calculation. One can directly use future worth calculations, but in here I have used present worth calculations. Either approach is fine and to improve your skills, it is recommended that you practice with both approaches.

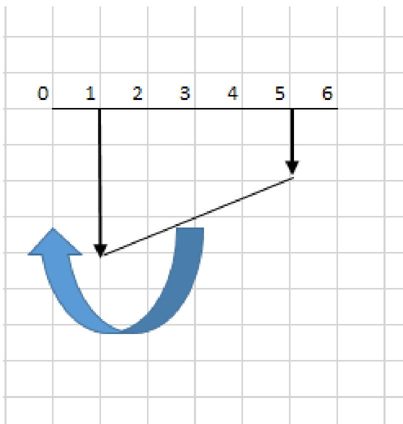


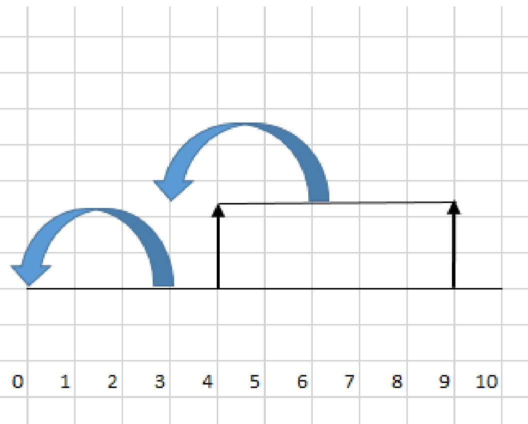
Present worth of individual payments is calculated using Individual payments at certain years using (P/F, 11.82%, n) factors. We have three individual transactions, one at year 0 (which is already at present worth), one at year 14, and one at year 20.

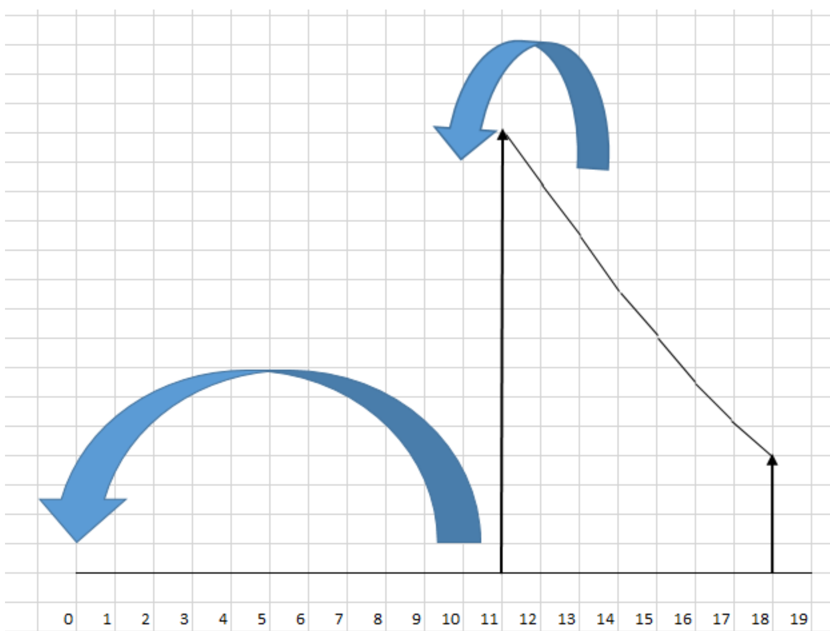
Present worth of annuities is calculated using (P/A, 11.82, n1) (P/F, 11.82%, n2). We have one such series between years 4 and 9 (6 years total).

Present worth of the arithmetic gradient series is calculated using (P/A, 11.82%, n) and (P/G, 11.82%, n). Since, the series begins at year 1, the resulting calculation will be automatically at year 0.

Present worth of the geometric gradient series has been calculated using the formula based on initial payment of the series and formula for (P/A, g, 11.82%, n). Since, the series begins at year 11, the resulting calculation need to be converted to year 0 using (P/F, 11.82%, n) factor.







Another instruction for the problem requires us not to use formula for the factors and use interpolation for the factors. Below are the calculations necessary to find the present worth followed by tables showing the interpolation results.

PW0 = - 4800

PW1 = - [3980(P/A, 11.82%, 5) -550(P/G, 11.82%, 5)]

PW1 = - [3980(3.6212) -550(6.4379)] = - 10871.53

PW2 = 3509 (P/A, 11.82%, 6) (P/F, 11.82%, 3) = 3509 (4.1328) (0.7153) = 10373.28

PW3 = A1 (P/ A1, g, i, n) (P/F, 11.82%, 10) = {A1 [1 – ((1 + g) / (1 + i))^n] / (i – g)} (P/F, 11.82%, 10)

(Note that the series is decreasing so, g = -16%)

PW3 = 15085 {[1- (0.84/1.1182)8] / (0.1182- (-0.16)} (0.3274) = 15952.49

PW4 = - 1848 (P/F, 11.82%, 14) = - 1848 (0.2095) = - 387.16

PW5 = 19870 (P/F, 11.82%, 20) = 19870 (0.1074) = 2134.04

PW = - 4800 - 10871.53 + 10373.28 + 15952.49 - 387.16 + 2134.04 = $12401.12

Period (n) = 3	Int. Rate %	Factor: (P/F, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	0.7312					
Target	11.82		1	-0.0194	0.82	0.7153	0.7152
High	12	0.7118					

Period (n) = 10	Int. Rate %	Factor: (P/F, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	0.3522					
Target	11.82		1	-0.0302	0.82	0.3274	0.3272
High	12	0.322					

Period (n) = 14	Int. Rate %	Factor: (P/F, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	0.232					
Target	11.82		1	-0.0274	0.82	0.2095	0.2093
High	12	0.2046					

Period (n) = 20	Int. Rate %	Factor: (P/F, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	0.124					
Target	11.82		1	-0.0203	0.82	0.1074	0.1071
High	12	0.1037					

Period (n) = 5	Int. Rate %	Factor: (P/G, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	6.624					
Target	11.82		1	-0.227	0.82	6.4379	6.4371
High	12	6.397					

Period (n) = 5	Int. Rate %	Factor: (P/A, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	3.6959					
Target	11.82		1	-0.0911	0.82	3.6212	3.6209
High	12	3.6048					

Period (n) = 6	Int. Rate %	Factor: (P/A, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	4.2305					
Target	11.82		1	-0.1191	0.82	4.1328	4.1325
High	12	4.1114					

Period (n) = 20	Int. Rate %	Factor: (F/P, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	8.0623					
Target	11.82		1	1.584	0.82	9.3612	9.3409
High	12	9.6463					

Note that the exact value of this present worth using formulas is $12383.38 (comparing to $12401.12 showing an insignificant error of 0.14%).

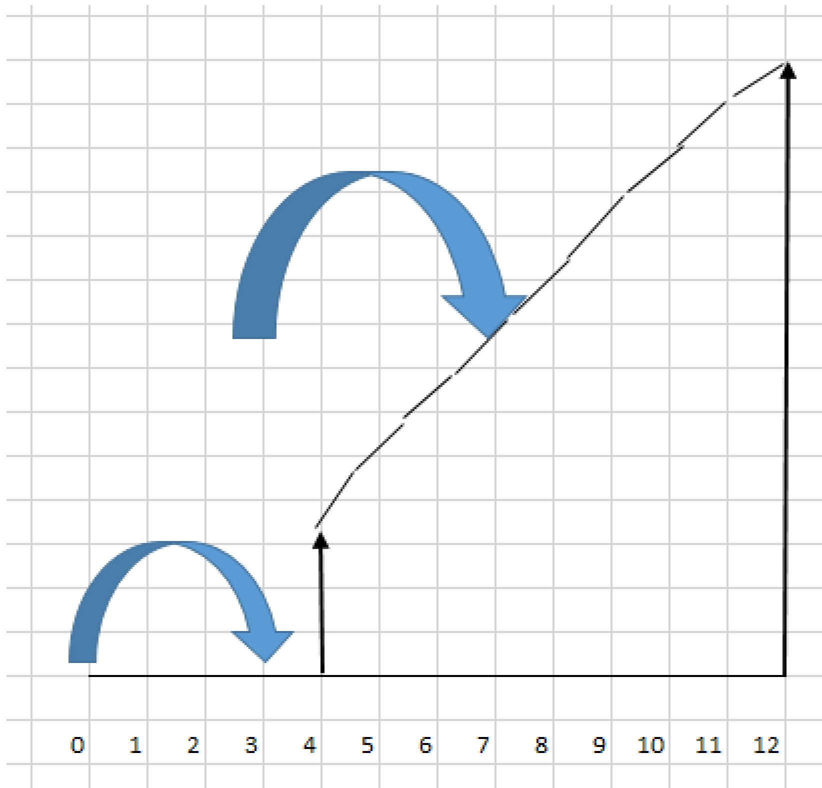
To answer part 2 question, we can find the equivalence of the current cash flow at year 3 and the use that to calculate the geometric gradient series that would yield such a present worth.

12401.12 (F/P, 11.82%, 3) = 12401.12 (1.3982) = 17339.25

Period (n) = 3	Int. Rate %	Factor: (F/P, i, n)	Rate Difference High - Low	Factor Difference	Rate Difference Target - Low	Target Factor Interpolation	Target Factor Formula
Low	11	1.3676					
Target	11.82		1	0.0373	0.82	1.3982	1.3982
High	12	1.4049					

Now, 17339.25 will be the equivalent of the geometric gradient series between years 4 to 12 with g=10% and i=11.82%. Then we solve for missing A1.

17339.25 = A1 (P/ A1, 10%, 11.82%, 9) = {A1 [1 – ((1 + g) / (1 + i))^{n}] / (i – g)}
A1 = 17339.25 / {[1 – (1.10 / 1.1182)^{9}] / 0.0182 } = 17339.25 / 7.544 = 2298.40



To calculate the rate of return for this cash flow we need to experiment with different rate of return and find the interest rate that achieves 0 for the present worth. Note that calculations for the present worth have been demonstrated above and does need to be demonstrated again. We set up a table for different interest rates and as soon as present worth sign changes, we limit the range of interest rate.

We begin with the calculated value of present worth (I will use formulas for accuracy). For 11.82% we had a positive present worth of 12383.38. To reduce that we need to increase the interest rate. Since 12383.38 is large let's try 16% and more.

Interest rate	Present worth	sign
11.82%	12383.38	> 0
16%	4191.76	> 0
18%	1562.14	> 0
19%	469.48	> 0
20%	-499.50	< 0

Thus the rate of return is between 19% (with PW=469.48) and 20% (with PW=-499.50). We are looking to a rate of return with PW=0. Performing interpolation:

- 499.50 - 469.48 = - 968.98 (change for 1%)
0 – 469.48 = -469.48 (target change)
(-469.48)/(-968.98) =0.4845
ROR = 18 + 0.4845 = 18.48%

Note that the real calculated value is 19.47% which very close to the calculated value.