### Assignment 1 Key

#### 70 Points

Assessment Goals: (Reading instructions; Following format guideline; Time management; Report quality; Calculation accuracy)

Show your work. No round down or up, use 2 decimals for dollar values and 4 decimals for factors.

#### **PROBLEM:**

Expenditures and income of a project over a 25-year period is provided in the table of the next page. Each student has his/her own section.

For the interest rate of 9% compounded annually, perform the following steps for the project:

1. Plot the cash flow (5 pts)

2. Calculate the present worth of the project using formulas for factors. (10 pts)

3. Calculate the present worth of the project using tables for factors and compare with answer in in part 2. Explain the reason for difference, if any. (10 pts)

4. Calculate future worth of the project at the end of year 25 using tables. (10 pts)

5. Calculate future worth of the project at the end of year 25 using the calculated value in part 3. (5 pts)

6. Calculate the equivalent value of the project at the end of year 14. (10 pts)

7. Using the value calculated in part 5 find the equivalent annual transaction for the whole cash flow over 25 years (5 points)

8. Find the present worth of the project when interest rate changes at the end of year 10 to 10% and at the end of year 20 to 12%. (15 points)

I will use the first one of the dataset for the solution key. Below is the plotted cash flow. Make sure that lines are proportional to the values. Here I used Excel and assumed each vertical grid is 500 and I marked them to make the counting easier. Then, for example, for the 1<sup>st</sup> transaction of -\$5,457, I plotted an arrow pointing downward almost 11 grids.



2. Calculate the present worth of the project using formulas for factors. (10 pts)

These transactions follow no specific patterns. So, the easiest approach is to find the present worth of each individual transaction and then add them all together. Since we are required to use the formulas, then the present worth of each individual transaction at the end of year *n* is:

$$P = F(1+i)^{-n}$$

Calculation example:

Transaction at the end of year 10 is \$6,292, which its present worth at year 0 is calculated as  $6,292(1+.0.0912)^{-10} = 6,292(0.4224) = 2,657.81$ . Similar calculations for the other numbers are presented in the table below totaling to PW = \$11,579.39

n	Transaction (\$)	(1+i)⁻ <sup>n</sup>	Present Worth (\$)
0	-\$5,457.00	1.0000	-\$5,457.00
1	\$4,330.00	0.9174	\$3 <i>,</i> 972.48
2	\$4,338.00	0.8417	\$3,651.21
3	\$0.00	0.7722	\$0.00
4	-\$5,453.00	0.7084	-\$3,863.04
5	\$261.00	0.6499	\$169.63
6	\$4,413.00	0.5963	\$2,631.33
7	\$0.00	0.5470	\$0.00
8	\$0.00	0.5019	\$0.00
9	\$6,599.00	0.4604	\$3 <i>,</i> 038.36
10	\$6,292.00	0.4224	\$2,657.81
11	-\$5,701.00	0.3875	-\$2,209.32
12	\$6,046.00	0.3555	\$2,149.56
13	\$0.00	0.3262	\$0.00
14	\$1,749.00	0.2992	\$523.38
15	\$0.00	0.2745	\$0.00
16	\$6,726.00	0.2519	\$1,694.08
17	\$0.00	0.2311	\$0.00
18	\$4,852.00	0.2120	\$1,028.59
19	-\$5,047.00	0.1945	-\$981.59
20	\$6,727.00	0.1784	\$1,200.30
21	\$0.00	0.1637	\$0.00
22	\$233.00	0.1502	\$34.99
23	\$0.00	0.1378	\$0.00
24	\$6,222.00	0.1264	\$786.49
25	\$4,761.00	0.1160	\$552.12

# 3. Calculate the present worth of the project using tables for factors and compare with answer in in part 2. Explain the reason for difference, if any. (10 pts)

Since we are required to use the tables, then the present worth of each individual transaction at the end of year *n* is:

$$P = F\left(\frac{P}{F}, i, n\right)$$

Looking at the table values we realize that factors exactly match the 4 digit calculation through the formulas. Thus the present worth will be the same.

i	9	INTEREST	
	Single Payment		
	Compound-	Present-	
	amount	worth	
n	Factor	Factor	
	to find F	to find P	
	given P	given F	
	F/P, i, n	P/F, i, n	
1	1.0900	0.9174	
2	1.1881	0.8417	
3	1.2950	0.7722	
4	1.4116	0.7084	
5	1.5386	0.6499	
6	1.6771	0.5963	
7	1.8280	0.5470	
8	1.9926	0.5019	
9	2.1719	0.4604	
10	2.3674	0.4224	
11	2.5804	0.3875	
12	2.8127	0.3555	
13	3.0658	0.3262	
14	3.3417	0.2992	
15	3.6425	0.2745	
16	3.9703	0.2519	
17	4.3276	0.2311	
18	4.7171	0.2120	
19	5.1417	0.1945	
20	5.6044	0.1784	
21	6.1088	0.1637	
22	6.6586	0.1502	
23	7.2579	0.1378	
24	7.9111	0.1264	
25	8.6231	0.1160	
26	9.3992	0.1064	
27	10 2451	0.0076	

Note that this is not true in general. The only reason that in this case the values for the factors are the same is because we restricted the values calculated through formulas to 4 digit decimal forms. For example, consider the same year 10 calculation where the factor value with more accurate number of decimals will be 0.422410806895689 instead of 0.4224 which is the rounded off number. However, the difference is not significant at all, usually less than a dollar.

## 4. Calculate future worth of the project at the end of year 25 using tables. (10 pts)

Do not forget that in here n is the number of years a transaction has to move to year 25 from the year it is located. For example, transaction at the end of year 10 has n=15 (i.e. 25-10=15) where transaction at year 0 has n=25. The calculation will result in Future worth of \$99,849.98.

Years to move forward	Transaction (\$)	(P/F, 9%, n)	Present Worth (\$)
25	-\$5,457.00	8.6231	-\$47,056.15
24	\$4,330.00	7.9111	\$34,254.99
23	\$4,338.00	7.2579	\$31,484.66
22	\$0.00	6.6586	\$0.00
21	-\$5,453.00	6.1088	-\$33,311.33
20	\$261.00	5.6044	\$1,462.75
19	\$4,413.00	5.1417	\$22,690.15
18	\$0.00	4.7171	\$0.00
17	\$0.00	4.3276	\$0.00
16	\$6,599.00	3.9703	\$26,200.05
15	\$6,292.00	3.6425	\$22,918.50
14	-\$5,701.00	3.3417	-\$19,051.19
13	\$6,046.00	3.0658	\$18,535.85
12	\$0.00	2.8127	\$0.00
11	\$1,749.00	2.5804	\$4,513.17
10	\$0.00	2.3674	\$0.00
9	\$6,726.00	2.1719	\$14,608.15
8	\$0.00	1.9926	\$0.00
7	\$4,852.00	1.8280	\$8,869.65
6	-\$5,047.00	1.6771	-\$8,464.32
5	\$6,727.00	1.5386	\$10,350.32
4	\$0.00	1.4116	\$0.00
3	\$233.00	1.2950	\$301.74
2	\$0.00	1.1881	\$0.00
1	\$6,222.00	1.0900	\$6,781.98
0	\$4,761.00	1.0000	\$4,761.00

FW= \$99,849.98

# 5. Calculate future worth of the project at the end of year 25 using the calculated value in part 3. (5 pts)

Present worth calculated in Part 3 was \$11,579.39. Either formula or factor from the table will result in the same value calculated in Part 4. F = P (F/P, 9%, 25) = (\$11,579.39) (8.6231) = \$99,849.98

#### 6. Calculate the equivalent value of the project at the end of year 14. (10 pts)

This is a combination of both present worth and future worth calculation. All transactions occurring BEFORE year 14 need to be brought forward to year 14 (that is future worth calculation) and all occurring AFTER year 14 need to move backward to year 14 (present worth calculation). For example, for transaction at the end of year 9 (\$6,599) we need to find its future worth with n=5 (from 9 to 14) and for transaction at the end of year 20 (\$6,727) we need to find its present worth with n=5 (from 19 to 14). The equivalent of the cash flow at year 14 is 16,162.52 + 22,526.62 = \$38,695.15 (Note that you have to make sure not to count the year 14 transaction twice.). Below are the calculations:

Years to move backward	Transaction (\$)	(1+i)⁻ <sup>n</sup>	Present Worth (\$)
0	\$1,749.00	1.0000	\$1,749.00
1	\$0.00	0.9174	\$0.00
2	\$6,726.00	0.8417	\$5,661.14
3	\$0.00	0.7722	\$0.00
4	\$4,852.00	0.7084	\$3,437.28
5	-\$5,047.00	0.6499	-\$3,280.20
6	\$6,727.00	0.5963	\$4,011.09
7	\$0.00	0.5470	\$0.00
8	\$233.00	0.5019	\$116.93
9	\$0.00	0.4604	\$0.00
10	\$6,222.00	0.4224	\$2,628.24
11	\$4,761.00	0.3875	\$1,845.04

#### \$16,168.52

These are the transactions in year 14 and after whose present worth at year 14 results in \$16,168.52.

move forward	Transaction (\$)	(P/F, 9%, n)	Present Worth (\$)
14	-\$5,457.00	3.3417	- \$18,235.80
13	\$4,330.00	3.0658	\$13,274.93
12	\$4,338.00	2.8127	\$12,201.34
11	\$0.00	2.5804	\$0.00
10	-\$5,453.00	2.3674	۔ \$12,909.23
9	\$261.00	2.1719	\$566.86
8	\$4,413.00	1.9926	\$8,793.18
7	\$0.00	1.8280	\$0.00
6	\$0.00	1.6771	\$0.00
5	\$6,599.00	1.5386	\$10,153.38
4	\$6,292.00	1.4116	\$8,881.67
3	-\$5,701.00	1.2950	-\$7,382.96
2	\$6 <i>,</i> 046.00	1.1881	\$7,183.25
1	\$0.00	1.0900	\$0.00

FW= \$22,526.62

## 7. Using the value calculated in part 5 find the equivalent annual transaction for the whole cash flow over 25 years (5 points)

In Part-5 we calculated the future worth of the cash flow. So we will use (A/F. i, n) factor to convert that into annuity over the whole period. Using table for 9% we have:

A = F (A/F, 9%, 25) = 99849.98 (0.0118) = \$1178.23

## 8. Find the present worth of the project when interest rate changes at the end of year 10 to 10% and at the end of year 20 to 12%. (15 points)

From year 0 to the end of year 10, interest rate is 9%. Beginning at the end of year 10 (start of year 11) to the end of year 20, the interest rate is 10% and for the remaining years it is 12%. To calculate the present worth we need to find the preset worth of individual transactions over the periods that have the same interest rate and then move them as a single item to year 0.

n	Transaction (\$)	(1+i) <sup>-n</sup>	Present Worth (\$)
0	-\$5,457.00	1.0000	-\$5,457.00
1	\$4,330.00	0.9174	\$3,972.48
2	\$4,338.00	0.8417	\$3,651.21
3	\$0.00	0.7722	\$0.00
4	-\$5,453.00	0.7084	-\$3,863.04
5	\$261.00	0.6499	\$169.63
6	\$4,413.00	0.5963	\$2,631.33
7	\$0.00	0.5470	\$0.00
8	\$0.00	0.5019	\$0.00
9	\$6,599.00	0.4604	\$3,038.36
10	\$6,292.00	0.4224	\$2,657.81

\$6,800.77

n	Transaction (\$)	(1+i) <sup>-n</sup>	Present Worth (\$)
0	\$0.00	1.0000	\$0.00
1	-\$5,701.00	0.9174	-\$5,230.28
2	\$6,046.00	0.8417	\$5,088.80
3	\$0.00	0.7722	\$0.00
4	\$1,749.00	0.7084	\$1,239.04
5	\$0.00	0.6499	\$0.00
6	\$6,726.00	0.5963	\$4,010.49
7	\$0.00	0.5470	\$0.00
8	\$4,852.00	0.5019	\$2,435.06
9	-\$5,047.00	0.4604	-\$2,323.78
10	\$6,727.00	0.1784	\$1,200.30
			C 440 C2

\$6,419.63

Note that the value at year 0 (actual year 10) was already considered in the previous table. Similarly, when we calculate the value for years 20-25, we are not going to include the transaction in year 20 since it is already included in the second table.

n	Transaction (\$)	(1+i) <sup>-n</sup>	Present Worth (\$)
0	\$0.00	0.1784	\$0.00
1	\$0.00	0.1637	\$0.00
2	\$233.00	0.1502	\$34.99
3	\$0.00	0.1378	\$0.00
4	\$6,222.00	0.1264	\$786.49
5	\$4,761.00	0.1160	\$552.12
			\$1,373.61

Resulting cash flow:



P = 6800.77 + [ 6419.63 + 1373.61 (P/F, 10%, 10) ] (P/F, 9%, 10)